Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
International Journal of Multiphase Flow 31 (2005) 1059–1062
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Letter to the Editor

Motion of a spherical liquid drop in a high-speed airstream

I will derive formulas for the motion of an initially motionless liquid drop of diameter d accelerated to the velocity U of a high speed air stream. The drop is accelerated by dynamic pressures embedded in the drag function correlation of Ortiz et al. (2003) for the drag coefficient

$$C_\mathrm{d} = 1.6 + 0.4 Oh^{0.08} We^{0.01}, \tag{1}$$

where

$$C_\mathrm{d} = \frac{2m\dot{V}}{\rho_\mathrm{a} A V^2}, \tag{2}$$

m is the mass of the drop, V is the air speed relative to the drop, $\dot{V}$ is the drop acceleration, ρ_a is the air density, A is the projected area, $We = \frac{d\rho_\mathrm{a}V^2/2}{\gamma}$ is the Weber number, d is the initial drop diameter, γ is surface tension, $Oh = \mu_\mathrm{L}/\sqrt{\rho_\mathrm{L}\gamma d}$ is the Ohnesorge number, μ_L is the liquid viscosity, ρ_L is the liquid density.

The correlation (1) depends weakly on viscosity through the Ohnesorge number and though it works also very well for viscoelastic drops, no viscoelastic parameter enters.

Joseph et al. (1999, JBB), studied the breakup of viscous and viscoelastic drops in a high speed air steam behind a shock wave in a shock tube. They made movies using a rotating drum camera giving one photograph every 5 μs. The first response of the drop after it is exposed to a high speed air stream is a flattening of the drop caused by pressure recovery. The drop also accelerates but does not at first move noticeably; hence V is the airspeed relative to a stationary drop. The displacement versus time curve obtained for the next stage in the breakup process is well fitted by a parabola indicating that constant acceleration is a good approximation for the early motion in the breakup process. The correlation (1) fits data from all the literature sources known to us and it predicts the acceleration from known quantities with reasonable accuracy. Drop acceleration is the single most important factor in the Rayleigh–Taylor (RT) instability breakup of liquid drops. In the experiments of JBB, the accelerations are huge, of the order 10^8 cm/s^2, and breakup initially commences with the formation RT waves on the front face of the drop. The agreement between the wave lengths predicted by RT instability using measured values of the acceleration and measured values of the wave lengths is outstanding.

0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2005.05.004

Less well understood are the events controlling breakup at later times; JBB note that there is a moderate drop-off of acceleration with time over the course of the 400 or 500 μs it takes to totally fragment the drop; they say that drops of the order of one millimeter are reduced to droplet clouds and possibly to vapor in times less than 500 μs: One idea about the fragmentation of the drop is that it ceases when the acceleration drops to a small value such that the drops are no longer at risk to RT instability. We think that once RT instability stops, further fragmentation of the drop ceases, and that this whole process is mainly controlled by the dynamic pressure. Here I will put a little meat on the bones by deriving a simple mathematical model of the acceleration of single spherical drop driven by dynamic pressure, ignoring drop deformation and breakup: We are looking at the scenario associated with a cascade; large drops break up by Rayleigh–Taylor (RT) instability when the acceleration due to the dynamic pressure of the relative velocity is large. The relative velocity decreases as the drop is accelerated to the free stream velocity. If the drop is large and the relative velocity is not too small, the drop will fragment into smaller drops which accelerate rapidly to a small relative velocity for which the RT instability is suppressed. Our goal is to find the drop sizes, free stream velocities and times for which the relative velocity is below the threshold for RT instability.

The main assumption of our model is that the acceleration of a spherical drop of any size is controlled by the dynamic pressure associated with the relative velocity. The correlation (1) already expresses this idea at the initial instant when $V = 0$, but it would have been better to write U instead of V for the stream velocity, where U is the free steam gas velocity and V is the actual velocity of the drop. The relative velocity is then $U - V$ and to keep things simple, we ignore the second term on the right of (1), writing

$$C_\mathrm{d} = 2m\dot{V}/\rho_\mathrm{a}A(U-V)^2 = 1.6, \tag{3}$$

where

$$V = 0 \quad @ \quad t = 0. \tag{4}$$

Ortiz et al. (2003) showed that for a spherical mass

$$\frac{m}{A} = \frac{2}{3}d\rho_\mathrm{L}, \tag{5}$$

so that (3) may be rewritten as

$$\dot{V} = \theta(U-V)^2, \tag{6}$$

where

$$\theta = 1.2l/d, \quad l = \rho_\mathrm{a}/\rho_\mathrm{L}. \tag{7}$$

The solution of (6) and (4) is

$$V(t) = \theta U^2 t(1+\theta U t)^{-1}. \tag{8}$$

We next consider a numerical example with values typical for breakup in the flow behind a Mach of 3 from experiments of Joseph et al. (1999). Data are given there for breakup in streams behind a shock moving at Mach 2 and 3. The RT analysis predicts breakup to very small waves; the more viscous the drop, the smaller is the wave length corresponding to maximum growth. The largest wavelength for which instability can occur is $\lambda_c = 2\pi\sqrt{\gamma/\rho_l a}$; this wavelength does not depend on the viscosity and increases with the square root of the acceleration. JBB estimate λ_c between 23 and 65 μm at a shock Mach of 3 and between 46 and 135 μm at a shock Mach of 2. The following values then are representative for breakup of the smallest drop which could undergo RT instability in a stream behind a shock moving at Mach 3:

$U = 7.6 \times 10^4$ cm/s, $d = 5 \times 10^{-3}$ cm is the size of drop so small that it will not fracture by Rayleigh–Taylor breakup, $\rho_a/\rho_L = 2.08 \times 10^{-3}$ is a value typical for water and air, $\theta U = 37{,}920$.

A 50 μm drop will accelerate to 95% of the stream velocity in about 5×10^{-4} s. This value is in good agreement with data for the cessation of breakup mentioned in the abstract and text of the paper by JBB. Larger drops that might be generated by RT instability of more viscous liquids take a longer time to speed up to the free stream. Equation (8) shows that a drop of diameter 100 μm takes twice as long to speed up.

We next integrate $V(t) = \mathrm{d}x/\mathrm{d}t$ to find the trajectory

$$x = Ut - \theta^{-1}\ln(1 + \theta Ut). \tag{9}$$

The spherical drop never quite accelerates to the free stream, like zeno, the difference between U and V gets smaller and smaller while the distance between the particle traveling with velocity U and the drop gets larger and larger.

By introducing

$$\tau = \theta Ut, \quad \chi = x\theta, \tag{10}$$

Eqs. (8) and (9) can be rewritten as

$$\frac{V}{U}(\tau) = \frac{\tau}{1+\tau}, \tag{11}$$

$$\chi(\tau) = \tau - \ln(1+\tau). \tag{12}$$

The dimensionless parameters V/U and χ are plotted in Fig. 1 as functions of τ.

Such long times are not observed because the large drops are fragmented into smaller drops by RT instabilities long before.

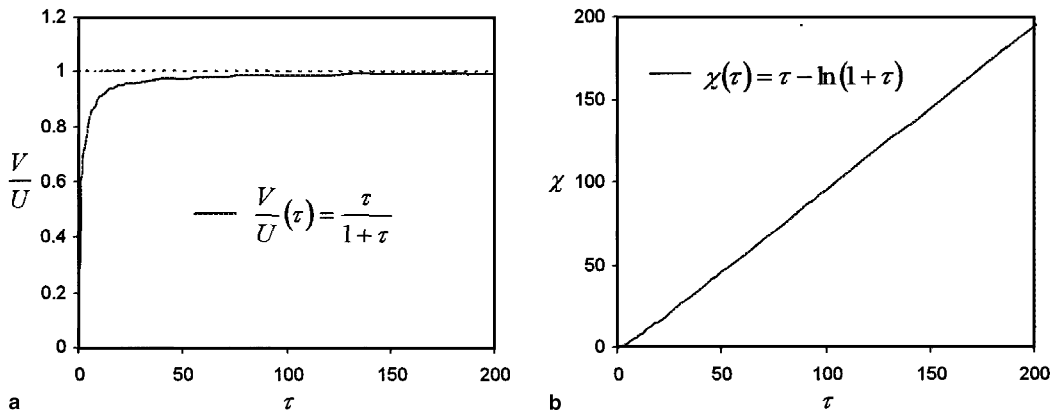

Fig. 1. Evolution of (a) V/U and (b) χ as functions of τ.

References

Joseph, D.D., Belanger, J., Beavers, G.S., 1999. Breakup of a liquid drop suddenly exposed to a high speed air stream. Int. J. Multiphase Flow 25, 1263–1303.

Ortiz, C., Joseph, D.D., Beavers, G.S., 2003. Acceleration of a liquid drop suddenly exposed to a high speed air stream. Int. J. Multiphase Flow 30, 217–224.

Daniel D. Joseph
Department of Aerospace Engineering and Mechanics
University of Minnesota, Minneapolis, MN 55455, USA
Tel.: +1 612 625 0309; fax: +1 612 626 1558
E-mail address: joseph@aem.umn.edu